A Preview of the New Economics of Knowledge

Martin J. Beckmann[a*]

[a]*Brown University and Technical University of Munich*

Abstract

Examples of knowledge as a subject for economic analysis are:
1. author and document evaluation by citations
2. joint authorship
3. size of research groups
4. knowledge as an input in production.

The optimal choice of products requiring various items of knowledge is made through an assessment of products for knowledge costs that can meet the Lindahl criterion of accounting.

JEL Classification: D83, A2

Keywords: Citations, joint authorship, research groups, knowledge inputs

An economics of knowledge was first proposed by Fritz Machlup in 1962. He was interested in "knowledge based industries" and their measurement. This was not pursued further by his students after his untimely death. The economics of knowledge has a certain vogue and is studied by contemporary Swedish economists.

Professor Åke Andersson and I are in fact writing a book on it and this is a "preview of coming attractions" with four illustrative examples.

I begin by stating a question that should not be asked in an economic study of knowledge, namely: what is knowledge? That should be left to the philosophers. We should ask instead: how can we measure knowledge for economic purposes?

Obviously only by means of proxy variables. How does a dean in an American university evaluate the competence (knowledge) of a candidate to be hired or promoted? Sometimes, it is said, by simply counting his / her publications. If that is too rough, he may count the number of pages or only the number of pages in refereed journals. Since citation indexes have arisen, a better measure might be the number of citations received (self-citations omitted). But to determine an article's significance should we not also consider the significance of the citing author or article? This could be done as follows:

[*] Charlotte Chen has kindly located for me the proper references to Frisch and Machlup. The Frisch production function was first pointed out to me by Professor Tönu Puu. I am grateful to both.

1. Let c_{ij} be the number of citation of i by j. In the case of documents (papers) $c_{ij} = 0$ or 1, in that of authors' o or a natural number. To estimate the significance v_i of i we propose proportionality:

$$v_i \sim \sum_j c_{ij} v_j$$

or

$$\lambda v_i = \sum_j c_{ij} v_j \tag{1}$$

the eigen-value equation. Not λ but the eigen vector (v_i) is of interest. Perron's Theorem (Perron, 1907) assures the existence of a positive $\lambda > v$ and a non-negative vector $(v_i) \geq 0$.

This procedure may be applied to evaluating, or at least ranking, not only documents and authors but also departments and scientific journals, for instance, "The 13 most cited journals in economics" (Beckmann and Persson, 1998). We note that *Econometrica*, often regarded to be the most prestigious of economic journals, ranks only in place three, after the AER and QJL in 1981–1996. The *Journal of Economic Literature*, to whom we might have submitted this brief paper, came out last. (So we didn't.)

Table 1

i	v_i
AER	944
QJE	589
ECA	570
RES	479
JPE	396
…	…
JEL	149

2. Let us turn to some spatial aspects of knowledge production. The location problems are challenging but do not have neat answers. Knowledge embedded in documents is easily transported (i.e. communicated) – the fastest and cheapest way being by email.

The knowledge embedded in a person is not. As Adam Smith remarked, "man is the most difficult luggage to transport".

There are thus economies of joint locations for knowledge production and distribution. That accounts for concentration in one or a few locations and also in organizations. Still, collaboration does also occur outside an organization and at a distance. This has been well documented and studied particularly for Scandinavian authors (Andersson and Persson, 1993) and for interaction between Sweden and the U.S., specifically in medicine and also in economics (and I have myself been so involved).

The data show that distance does matter. Let us see why and how.

Consider two authors writing a paper. Let their contribution (effort) be x and y. We postulate a production function with two inputs, where only one is needed, $f(x, y)$. For a collaboration to be fruitful, one must have:

$$f(x, y) > f(x, o) + f(o, y) \tag{2}$$

Or when working full time on this (for a while)

$$f(1, 1) > f(1, o) + f(o, 1)$$

denoting complementarily.

Let interaction at distance r take away time kr

$$f([1 - kr]x, [1 - kr]y) > f(x, y)$$

Or with linear homogeneity and $x = y = 1$

$$[1 - kr]\, f(1, 1) > f(1, 0) + f(0, 1)$$

$$kr < \frac{f(1,1) - f(0,1) - f(1,0)}{f(1,1)} = u \tag{3}$$

where the right hand side is a measure of the advantage of collaboration.

Now let u be considered a random variable, normally distributed: many additive random elements are involved. The emerging probability of collaboration is then.

$$Pr\ (\text{collab.}) = 1 - N(kr) \tag{4}$$

where N denotes the (standardized) normal distribution. Using the well-known approximation

$$N_{(x)} \doteq \frac{1}{1 + e^{-\alpha x}} \tag{5}$$

$$pr(collab.) = 1 - \frac{1}{1 + e^{-ak}} = \frac{1}{1 + e^{akr}} \doteq e^{-akr} \tag{6}$$

establishing an exponential distance effect (for longer distances), well known for other types of spatial interaction (Alan Wilson).

Let me mention some more locational aspects of knowledge. Knowledge distribution in the form of teaching is a market-oriented activity. At the elementary level it even sells a homogeneous product. There is no free entry and the profit motive is absent. (In fact we are dealing with central planning rather than a market economy.) The Löschiam Model does not fit here. Also in an urban context, the market areas should not take the shape of hexagons, but of (tilted) squares on a rectangular grid.

At the college level the monopolistic competition model seems applicable, even though the industry pretends not to be profit oriented. To the degree that the product is heterogeneous, cross-hauling prevails. Market areas can be defined only as domains of dominant market shares.

Production of basic knowledge is allied to teaching in "research universities" and thus falls into the location pattern of colleges. Since these are a (small) subset of colleges, the locations are fewer and spaced farther apart as centers of larger (sometimes national) market areas.

In applied research, the forces of attraction are existing research universities and clients. A Weber model is possible, but unlikely when clients may change. Industrial research laboratories are typically drawn to the parent firm's headquarters or they move to a university with strong presence in their field, preferably boasting a Nobel Prize winner.

A truly footloose knowledge producer is the independent writer of scientific textbooks (rare) or of outright fiction. Such authors can seek out the delights of scenic or rustic places or shunning the "idiotism of rural life" (Marx-Engels), go for the flesh pots of the metropolis.

Footloose are also the think tanks which may find a refuge in quiet and balmy surroundings but not too far from good schools and other needs of scholars' families.

Conventions are periodic fairs which typically rotate. Incidentally it has been calculated that the location that minimizes total travel cost for the members of the AEA is Pittsburg, PA (Siegried).

3. My third example concerns the structure of group production and the choice of output measures to identify a research group of optimal size. Such a group is not a team under a team leader but a voluntary, and perhaps temporary, association of independent researchers engaged in the pursuit of a common research topic.

Each one of the x members communicates with all the $x - 1$ others, and this costs time k for each pair. Total working time per person is thus:

$$1 - k(x - 1)$$

and group labor time is:

$$x[1 + k - kx].$$

The other input into the group's production is the combined group's knowledge assumed proportional to group size x. Assume a Cobb-Douglas production function scaled to yield an output of unity at group size 1.

$$Q(x) = \left(\left[1 + k - kx\right] x\right)^{\alpha} x^{\beta} \tag{7}$$

or

$$Q(x) = \left[1 + k - kx\right]^{\alpha} x^{\eta} \tag{7a}$$

$$\eta = \alpha + \beta$$

The rationale of groups is to reap economies of scale

$$\eta > 1 \tag{8}$$

With these assumptions the group production function (7) has initially increasing and the then decreasing returns, i.e. is at first convex and then concave.

For $\alpha = 1$, $\eta = 2$, it is, in fact the FRISCH production function

$$F(x) = ax^2 - x^3 \tag{7b}$$

introduced in his famous text Production (in Norwegian) from 1926, as a convex / concave type.

Consider in fact the second derivative

$$\begin{aligned} Q''(x) = {} & \alpha(\alpha-1)k^2[1+k-kx]^{\alpha-2}x^{\eta} \\ & -2\alpha\eta k[1+k=kx]^{\alpha-1}x^{\eta-1} \\ & +\eta(\eta-1)[1+k-kx]^{\alpha}x^{\eta-2} \end{aligned} \tag{9}$$

For small $x << 1$ the lowest power of x dominates, so that $Q'' > 0$, provided $\eta > 1$, as assumed.

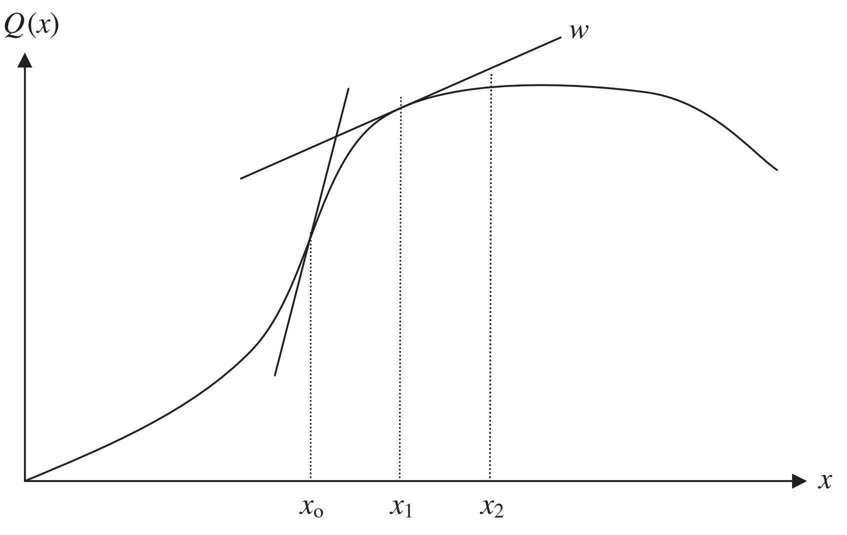
For large x the highest power x^{η} dominates, which is negative for $0 < \alpha < 1$, and also at the point of maximal output with

$$0 = Q'(x) \sim -k\alpha x + \eta\,(1 + k - kx)$$

yielding (Figure 1)

$$x_2 = \frac{\eta}{\alpha+\eta} \cdot \frac{1+k}{k} \tag{10}$$

Figure 1



Should group size for maximal output be the objective? It was, in the GM research laboratories. (But there a team model might be more appropriate.)

When the choice is up to the members themselves, as would be natural, the goal might be maximal individual (i.e. marginal) productivity: at the turning point x_0.

The socially optimal objective is group output minus the opportunity cost of group members' time

$$Q(x) - wx \tag{11}$$

maximized at the point x_1 of meeting a tangent with slope w in Figure 1 but to the right of x_0.

Raising η or lowering k moves the three points to the right, as economic intuition suggests.

4. As a final example consider the uses of knowledge as an input into production. Assume for simplicity a linear technology for a firm making products $i = 1, \dots n$. Material inputs and labor are not considered explicitly, only profit margins g_i (before knowledge costs) and knowledge input coefficients a_{ik} for knowledge of type k required in the production of i.

Now knowledge use has two significant aspects. Knowledge items are "lumpy", used either in full or not at all. Secondly, knowledge is non-rivalrous, i.e. can be used simultaneously in several activities. Moreover, knowledge is not used up, but remains available, but that is irrelevant here.

Formally this means that knowledge is a zero-one variable to the firm. Allocation of resources becomes a mixed integer-linear program.

Assume that demand is given and standardized to be one unit. The firm chooses production x_i of products i and uses y_k of knowledge items k to maximize net profits

$$\max \sum_i g_i x_i - \sum_k p_k y_k$$

with given knowledge costs p_k and restrictions:

$$0 \le x_i \le 1$$

$$y_i \ge a_{ik} x_i$$

This mixed integer linear program can in fact be solved as a linear program with output and knowledge utilization variables x_j and y_k treated as continuous non-negative variables.

The LaGrange function is then

$$\sum_i g_i x_i - \sum_k p_k y_k + \sum_{i,\,k} \lambda_{ik}(y_k - a_{ik} x_i) + \sum_i \mu_i (1 - x_i)$$

and the efficiency conditions of the linear program are:

$$x_i \begin{Bmatrix} = \\ \geq \end{Bmatrix} 0 \Leftrightarrow g_i \begin{Bmatrix} < \\ = \end{Bmatrix} \sum_k a_{ik}\,\lambda_{ik} + \mu_i \tag{12}$$

$$y_k \begin{Bmatrix} = \\ \geq \end{Bmatrix} 0 \Leftrightarrow p_k \begin{Bmatrix} > \\ = \end{Bmatrix} \sum_i \lambda_{ik} \tag{13}$$

$$\lambda_{ik} \leq \begin{Bmatrix} = \\ \geq \end{Bmatrix} 0 \Leftrightarrow y_k \begin{Bmatrix} > \\ = \end{Bmatrix} a_{ik}\, x_k \tag{14}$$

$$\mu_i \begin{Bmatrix} = \\ \geq \end{Bmatrix} 0 \Leftrightarrow x_i \begin{Bmatrix} < \\ = \end{Bmatrix} 1 \tag{15}$$

(13) and (14) are the Lindahl criterion for common use, of available knowledge, here by the production processes x_i. Mathematically this is a situation comparable to the (linear) Assignment Problem (Koopmans and Beckmann, 1957) where an Integer Programming Problem was reduced to a Linear Program and yielded an economic interpretation.

As an example, consider seven production processes i using three items of knowledge k in the seven possible combinations of Table 2, listing the knowledge input coefficient a_{ik}.

In Table 3 the assessments λ_{ik} of production i for use of k are shown. They satisfy the profitability and Lindahl conditions (2), (3) with $\mu_i \equiv 0$.

All products can be produced. There is no surplus since knowledge costs absorb all profits:

$$\sum_i g_i = 16 = \sum_k p_k\,.$$

Any increase in the price of some k would eliminate not just the products i using this k but all others as well since they needed the sharing of their own knowledge costs by those directly affected products i.

Table 2

$\boldsymbol{a_{ik}}$

i \ k	1	2	3	g_i
1	1			1
2		1		2
3			1	3
4	1	1		4
5		1	1	3
6	1		1	2
7	1	1	1	1
p_k	4	6	6	

Table 3
λ_{ik}

i \ k	1	2	3	g_i
1	1			1
2		2		2
3			3	3
4	2	2		4
5		1	2	3
6	1		1	2
7		1		1
p_k	4	6	6	

These four problems I have chosen more for their technical appeal rather than their inherent economic significance. In conclusion let me list some economic problems that cry out to be considered:

- what knowledge should be continued i.e. taught?
- what new knowledge should be pursued i.e. researched?
- what knowledge should be published, what should be kept as private property?
- why and how should basic research be financed?
- how can researchers be motivated?
- what is the role of markets in knowledge production?
- what is the case or government action in knowledge affairs?

Some topics in the economics of knowledge have been studied in other contexts, e.g. the impact of research and development on economic growth, but the reverse question should also be asked: how is knowledge production (research) affected by economic growth?

Other topics such as interaction in knowledge networks have been pursued by historians of science and sociologists. Just as it is said that “war is too important to be left to the generals”, I say that “knowledge is too important to be left out of economics”.

References

Andersson, Å and O. Persson, 1993, “Networking Scientists”, *Annals of Regional Science* 27, 11–21.

Beckmann, M. and O. Persson, 1998, “The 13 Most Cited Journals in Economics”, *Scientometrics* 42, 2, 267–271.

Frisch, R., 1926a, “Memorandum from the University Institute of Economics, Oslo”, unpublished.

Frisch, R., 1926b, “Kvantitativ formulering av den teroretiske økonomikks lover (Quantitative formulation of the laws of economic theory)”, *Statsøkonomisk Tidsskrift*, 299–334.

Koopmans, T.C. and M. Beckmann, 1957, “Assignment Problems and the Location of Economic Activities”, *Econometrica*, 25(1), 53–76.

Machlup, F., 1962, *The Production and Distribution of Knowledge in the United States* (Princeton: Princeton University Press).

Perron, O., 1907, “Zur Theorie der Marizen”, *Mathmatische Annalen* 64, 248–263.

Wilson, A., 1970, *Entropy in Urban and Regional Modelling* (London: Pion).